CLAREMONT CENTER
for MATHEMATICAL SCIENCES

Fall 2018 Poster Session

Wednesday, September 19, at 4:15 PM

Argue Auditorium, Pomona College

Computing Eigenmodes of the Laplace-Beltrami Operator by Using Radial Basis Functions

by

Vladimir Delengov

Chiu-Yen Kao

Claremont Graduate University

Abstract: In this work, numerical approaches based on meshless methods are proposed to obtain eigenmodes of the Laplace-Beltrami operator on manifolds, and their performance is compared against existing alternative methods. Radial Basis Function (RBF)-based methods allow one to obtain interpolation and differentiation matrices easily by using scattered data points. We derive expressions for such matrices for the Laplace-Beltrami operator via so-called Reillys formulas and use them to solve the respective eigenvalue problem. Numerical studies of proposed methods are performed in order to demonstrate convergence on simple examples of one-dimensional curves and two-dimensional surfaces. Prospective extensions of the methods include application to problems with boundary conditions and incorporating a multi-layer approach in order to improve accuracy. The latter is justified by an asymptotic expansion of eigenvalues of Laplace operator on a thin ring and a thin shell.

Covariance-based Dissimilarity Measures Applied to Clustering Wide-sense Stationary Ergodic Processes

by

Nan Rao

Qidi Peng, Ran Zhao

Claremont Graduate University

Abstract: We introduce a new unsupervised learning problem: clustering wide-sense stationary ergodic stochastic processes. A covariance-based dissimilarity measure and consistent algorithms are designed for clustering offline and online data settings, respectively. We also suggest a formal criterion on the efficiency of dissimilarity mea- sures, and discuss of some approach to improve the efficiency of clustering algorithms, when they are applied to cluster particular type of processes, such as self-similar processes with wide-sense stationary ergodic increments. Clustering synthetic data sampled from fractional Brownian motions is provided as an example of application.

Generalized Covariation of Symmetric -stable Distributions

by

Yujia Ding

Qidi Peng

Claremont Graduate University

Abstract: We introduce a new measure of dependency between coordinates of a symmetric α-stable random vector that we call the generalized covariation. We show that via a new type of fractional derivative and fractional Taylor expansion, these covariations can be used to fully characterize the joint distribution of bi-variate symmetric α-stable variables.

Learning to Fail: Predicting Fracture Evolution in Brittle Materials using Recurrent Graph Convolutional Neural Networks

by

Yadong Ruan, Zhengming Song

Max Schwarzer, Bryce Rogan, Diana Lee, Allon G. Percus, Viet T. Chau, Gowri Srinivasan, Hari Viswanathan, Bryan Moore

Claremont Graduate University

Abstract: Understanding dynamic fracture propagation is essential to predicting how brittle materials fail. Various mathematical models and computational applications have been developed to predict fracture evolution and coalescence, including finite-discrete element methods such as the Hybrid Optimization Software Suite (HOSS). While such methods achieve high fidelity results, they can be computationally prohibitive: a single simulation takes hours to run, and thousands of simulations are required for a statistically meaningful ensemble. We propose a machine learning approach that, once trained on data from HOSS simulations, can predict fracture growth statistics within seconds. Our method uses deep learning, exploiting the capabilities of a graph convolutional network to recognize features of the fracturing material, along with a recurrent neural network to model the evolution of these features. In this way, we simultaneously generate predictions for qualitatively distinct material properties. Our prediction for total damage in a coalesced fracture, at the final simulation time step, is within 3% of its actual value, and our prediction for total length of a coalesced fracture is within 2%. We also develop a novel form of data augmentation that compensates for the modest size of our training data, and an ensemble learning approach that enables us to predict when the material fails, with a mean absolute error of approximately 15%.

Tribracket Modules

by

Yingqi Shi

Deanna Needell, Sam Nelson, Yingqi Shi

Claremont Mckenna College

Abstract: *Niebrzydowski tribrackets* are ternary operations on sets satisfying conditions obtained from the oriented Reidemeister moves. Tribracket coloring over oriented knots and links defines a counting invariant of oriented knots and links. In this project, we **enhance** the tribracket counting invariant by defining *tribracket modules* over any given tribracket X, with axioms motivated by Reidemeister moves under a coloring scheme introduced as *secondary colorings.*

Equiangular Tight Frames and Corresponding Lattices

by

Jessie Xin

Lenny Fukshansky

Claremont Mckenna College

Abstract: An equiangular tight frame is a family of pairwise equiangular vectors in a Euclidean space that satisfies Parseval's identity. The integral span of the vectors of an equiangular tight frame is a lattice if and only if the inner product of the frame vectors is rational. We show that the symmetric group of such equiangular tight frame is always a subgroup of the automorphism group of its corresponding lattice. We also look into a way to construct a certain type of lattice-generating equiangular tight frame from a cyclic group and study some interesting properties of such equiangular tight frame and its corresponding lattice.

Quandle Coloring Quivers

by

Karina Cho

Sam Nelson

Harvey Mudd College

Abstract: We consider a quiver structure on the set of quandle colorings of an oriented knot or link diagram. This structure contains a wealth of knot and link invariants and provides a categorification of the quandle counting invariant, i.e., giving the set of quandle colorings the structure of a category which is unchanged by Reidemeister moves. We derive some new enhancements of the counting invariant from this quiver structure and show that the enhancements are proper with explicit examples.

Effective Bounds for Traces of Maass-Poincar Series

by

Havi Ellers

Harvey Mudd College

Abstract: An important problem in Number Theory is that of bounding the Fourier coefficients of modular forms. Work of Duke and Jenkins, and Miller and Pixton, shows that the generating functions for traces of Maass-Poincar series appear as holomorphic parts of certain half-integral weight weakly holomorphic modular forms. Our goal was to find an explicit upper bound for traces of Maass-Poincar series, which can then be used to help bound the Fourier coefficients of the aforementioned half-integral weight modular forms.

Personal Beliefs and Election Forecasts

by

Harry Bendekgey

Pomona College

Abstract: Data science is not a purely objective pursuit. When trying to forecast elections, because there are so few data points, (only about a hundred years of "modern" elections) it is important that predictors be rooted in theory instead of, say, trying to predict a winner based on how many syllables in their first name. But whose theory do you trust? Forecasters disagree on what will happen in the 2018 house elections. Will it be a blue wave, or are Democrats just a modest favorite to take the house? I compare the academic states-of-the-art to make clear what it means for some of them to be right this year and not others. Further, I propose and implement methods for encoding your own personal beliefs into a forecast, and see what that means, district-by-district.

Hemoglobin Response to Higher Order Gene Interactions

by

Sylvia Akueze Nwakanma

Lillian Gonzales, Rosa Garza

Pomona College

Abstract: Mutations, or alterations in genetic sequencing, contribute to variations in phenotype, yet the extent of contribution is often confounded when multiple mutations are being considered. With the growing amount of genomics data, there is a need for efficient and effective methods of analyzing mutation interactions. In this paper, we use spectral analysis to linearly decompose a genomics data set into its principle components. This approach allows us to combinatorially analyze higher order interactions between mutations and study their effects on the hemoglobin levels of the individuals in our data. A comparison between our method and classic multilinear regression models was then performed to highlight the advantages that the spectral analysis approach has in identifying significant mutation interactions.

Exploring Cell Differentiation Trajectories through Data Reduction

by

Gianna Wu

Michelle Li

Pomona College

Abstract: Understanding the processes of cell differentiation and proliferation is crucial to developing a deeper knowledge of cell-based diseases like cancer. New high-dimensional biological data, particularly those provided by genomic analysis and flow and mass cytometry, are making it clear that clustering cell phenotypes into distinct populations does not reflect the indistinct boundaries between cell states. Recent developments in cytometry now allow the tracking of cell development over time in terms of increasingly large numbers of phenotype variables simultaneously, but the mathematical tools and frameworks to describe the evolution of these states are not yet fully developed. The main objective of this REU project is to provide a beginner's guide to dimension reduction techniques, especially nonlinear dimension reduction. These techniques include Principal Component Analysis (PCA), Diffusion Mapping, t-distributed Stochastic Neighborhood Embedding (t-SNE), and Uniform Manifold Approximation and Projection (UMAP). We explore these approaches and apply them to biological data sets to help determine which combinations of approaches are most effective at providing the critical information needed for characterizing cell differentiation, with the long-term goal of developing new mathematical descriptions of a continuous phenotype space.